Direct Signals of Low Scale Gravity at e^+e^- Colliders

Kingman Cheung
Department of Physics, University of California, Davis, CA 95616 USA

Wai-Yee Keung
Department of Physics, University of Illinois at Chicago, Chicago IL 60607-7059
(February 1, 2008)

Gravity can become strong at the TeV scale in the theory of extra dimensions. An effective Lagrangian can be used to describe the gravitational interactions below a cut-off scale. In this work, we study the associated production of the gravitons with a Z boson or a photon at e^+e^- colliders of energies of LEPII to the Next Linear Colliders (NLC) ($\sqrt{s} = 0.25 - 1.5$ TeV) and calculate the sensitivity to the new interactions. We also obtain the limit on the cut-off scale using the present data from LEPII.

I. INTRODUCTION

Recent advances in string theories suggest that a special 11-dimension theory (dubbed as M theory) [1] may be the theory of everything. Impacts of M theory on our present world can be studied with compactification of the 11 dimensions down to our $3+1$ dimensions. The path of compactification is, however, not unique. One popular path is to first compactify the 11 dimensions down to 5 dimensions [2]. In this 5-dimensional world, the standard model particles live on a brane ($3+1$ dim) while there are other fields, like gravity and super Yang-Mill fields, live in the bulk. A novel mechanism to break the supersymmetry (SUSY) is offered in this picture, in which a hidden sector lives on another brane in this 5-dimensional world. This brane is entirely separated from the standard model (SM) brane. While SUSY is broken in this hidden brane by any means, the SUSY breaking is communicated to the SM brane via the interactions of the fields in the bulk. Large number of studies in this area have been proposed [3]. Apart from the above, radical ideas like TeV scale string theories were also proposed [4].

Inspired by string theories a simple but probably workable solution to the gauge hierarchy was recently proposed by Arkani-Hamed, Dimopoulos and Dvali (ADD) [5]. They assumed the space is $4+n$ dimensional, with the SM particles living on a brane. While the electromagnetic, strong, and weak forces are confined to this brane, gravity can also propagate in the extra dimensions. To solve the gauge hierarchy problem they proposed the "new" Planck scale M_S is of the order of TeV in this picture with the extra dimensions of a very large size R. The usual Planck scale $M_G = 1/\sqrt{G_N} \sim 1.22 \times 10^{19}$ GeV is related to this effective Planck scale M_S by using Gauss's law:

$$R^n \, M_S^{n+2} \sim M_G^2 \,. \tag{1}$$

For $n=1$ it gives a large value for R, which is already ruled out by gravitational experiments. On the other hand, $n=2$ gives $R \lesssim 1$ mm, which is in the margin beyond the reach of present gravitational experiments.

The graviton including its excitations in the extra dimensions can couple to the SM particles on the brane with a strength of $1/M_S$ at short distances (instead of $1/M_G$), and thus the gravitation interaction becomes comparable in strength to weak interaction at TeV scale. Hence, it can give rise to a number of phenomenological activities testable at existing and future colliders [6–10]. So far, studies show that there are two categories of signals: direct and indirect. The indirect signal refers to exchanges of gravitons in the intermediate states, while direct refers to production or associated production of gravitons in the final state. Since gravitons interact weakly with detectors, they will escape detection and give rise to missing energies. Thus, the logical signal to search for would be the associated production of gravitons with other SM particles. At e^+e^- colliders, the best signals would be the associated production of graviton with a Z boson, a photon, or a fermion pair. The production of graviton and photon at LEPII has been studied in Ref. [9]. In this work, we shall study the associated production of graviton with a Z boson or a photon at e^+e^- colliders of energies from LEPII to about 1.5 TeV.

The detection of photon will be direct while for the Z boson its decay products, a lepton pair or a quark pair, need to be detected. The branching ratio of the Z boson into visible products is about 80% (the decay of Z into neutrinos will not be useful because the final state would then be all missing energies.) Therefore, the signature will be a lepton pair or a quark pair of the Z mass with a large missing energy (or a photon with missing energy for γG production.) In contrary to the background, the recoil mass spectrum will not show a particular resonance because the mass spectrum of the graviton excitations is almost a continuous one. Recent analyses were performed by L3, ALEPH, and DELPHI [11] at LEPII in search for ZH production with the Higgs boson decaying into invisible particles. This signature is similar to our signal of interest. Thus, we shall also use these data to constrain the new cut-off scale M_S. In order

to have a handle on the feasibility of our signal, we choose ZH production as a bench-mark for comparison with the cross section of ZG. The major background comes from $e^+e^- \to Z\ (\gamma)\ \nu_i\bar{\nu}_i$ $(i=e,\mu,\tau)$, of which ν_μ and ν_τ come mainly from ZZ (γZ) pair production while ν_e also has contributions from W-exchange diagrams.

The organization of the paper is as follows. In the next section, we shall give the details of the calculations. In Sec. III, we shall give numerical results and derive possible limits that can be reached by the NLC. In Sec. IV, we use the data from LEPII on the search for ZH production with the Higgs boson decaying invisibly and the data on the production of a single photon with missing energy to constrain the cut-off scale. We shall then conclude in Sec. V.

II. CALCULATIONS

We concentrate on the spin-2 component of the Kaluza-Klein (KK) states, which are the excited modes of graviton in the extra dimensions. The spin-1 and spin-0 components are less interesting phenomenologically. We follow the convention in Ref. [8]. There are four contributing Feynman diagrams for the process $e^+e^- \to ZG$: one 4-point vertex diagram and the other three diagrams are obtained by attaching the graviton to each leg of e^+, e^-, and Z. The amplitudes for $e^-(p_1)e^+(p_2) \to Z(k_1)G(k_2)$ are given by:

$$\begin{aligned}
\mathcal{M}_1 &= \frac{g\kappa}{2\cos\theta_W}\,\frac{1}{s-m_Z^2}\,\bar{v}(p_2)\gamma_\beta(g_v-g_a\gamma^5)u(p_1)\ \left(\eta^{\alpha\beta}-\frac{(k_1+k_2)^\alpha(k_1+k_2)^\beta}{m_Z^2}\right)\\
&\quad\times\Big[-2k_1\cdot k_2\,\epsilon^\nu(k_1)\epsilon_{\alpha\nu}(k_2)+2\epsilon^\nu(k_1)\epsilon_{\mu\nu}(k_2)k_1^\mu k_{1\alpha}\\
&\qquad -2\epsilon_\alpha(k_1)\epsilon_{\mu\nu}(k_2)k_1^\mu k_1^\nu+2k_2\cdot\epsilon(k_1)\epsilon_{\alpha\mu}(k_2)k_1^\mu\Big]\ , \qquad (2)\\
\mathcal{M}_2 &= \frac{g\kappa}{2\cos\theta_W}\,\frac{1}{(k_2-p_2)^2}\bar{v}(p_2)\gamma^\mu(\not{k}_2-\not{p}_2)\not{\epsilon}(k_1)(g_v-g_a\gamma^5)u(p_1)\ p_2^\nu\ \epsilon_{\mu\nu}(k_2)\ , \qquad (3)\\
\mathcal{M}_3 &= -\frac{g\kappa}{2\cos\theta_W}\,\frac{1}{(p_1-k_2)^2}\bar{v}(p_2)\not{\epsilon}(k_1)(g_v-g_a\gamma^5)\,(\not{p}_1-\not{k}_2)\gamma^\mu u(p_1)\ p_1^\nu\ \epsilon_{\mu\nu}(k_2) \qquad (4)\\
\mathcal{M}_4 &= \frac{g\kappa}{2\cos\theta_W}\ \bar{v}(p_2)\gamma^\nu(g_v-g_a\gamma^5)\,u(p_1)\ \epsilon^\mu(W)\,\epsilon_{\mu\nu}(G)\ . \qquad (5)
\end{aligned}$$

where $\kappa=\sqrt{16\pi G_N}$, g_v and g_a are the vector and axial-vector coupling of Z to electron.

We have used the REDUCE program to evaluate the square of the sum of amplitudes. The spins of the Z boson and the graviton are summed using these formulas:

$$\begin{aligned}
\sum_s \epsilon_\mu^s(k_1)\,\epsilon_\nu^{s*}(k_1) &= -\eta_{\mu\nu}+\frac{k_{1\mu}k_{1\nu}}{m_Z^2}\ ,\\
\sum_s \epsilon_{\mu\nu}^s(k_2)\,\epsilon_{\rho\sigma}^{s*}(k_2) &= \tfrac{1}{2}B_{\mu\nu,\rho\sigma}(k_2)\ \ , \qquad (6)
\end{aligned}$$

where $B_{\mu\nu,\rho\sigma}(k_2)$ is given by [8]

$$\begin{aligned}
B_{\mu\nu,\rho\sigma}(k_2) &= P_{\mu\rho,\nu\sigma}(k_2)+P_{\mu\sigma,\rho\nu}(k_2)-\tfrac{2}{3}P_{\mu\nu,\rho\sigma}(k_2)\ ,\\
P_{\mu\nu,\rho\sigma}(k_2) &= (\eta_{\mu\nu}-k_{2\mu}k_{2\nu}/m^2)(\eta_{\rho\sigma}-k_{2\rho}k_{2\sigma}/m^2)\ . \qquad (7)
\end{aligned}$$

Here m denotes the mass of the KK state. The spin-averaged amplitude squared for the process $e^+e^- \to ZG$ is given by

$$\begin{aligned}
\overline{\sum}|\mathcal{M}|^2 = \frac{g^2\kappa^2(g_v^2+g_a^2)}{48\cos^2\theta_W u^2t^2(s-m_Z^2)^2}\Big\{&8m_Z^6tu[3m^2(m^2-t-u)+4tu]\\
&+2m_Z^4tu[27m^6-42m^4(t+u)+15m^2(t^2+u^2)+80m^2tu-28(t^2u+tu^2)]\\
&+m_Z^2[3m^8(-t^2-u^2+12tu)+6m^6(t^3-12(t^2u+tu^2)+u^3)+3m^4(-t^4+14t^3u+62t^2u^2+14tu^3-u^4)\\
&\quad+6m^2(-t^4u-23(t^3u^2+t^2u^3)-tu^4)+36(t^4u^2+t^2u^4)+52t^3u^3]\\
&+3ut(-m^2+t+u)[-m^4+m^2(t+u)-4tu][2m^4-2m^2(t+u)+t^2+u^2]\Big\}\ . \qquad (8)
\end{aligned}$$

To obtain the total cross section we have to sum over all discrete KK states with $m_k = 2\pi k/R$ for all m_k below $\sqrt{s} - m_Z$. Since the mass spacing of these KK states is much smaller than any physical scales in the problem, it is convenient to convert the discrete sum on k to an integral over m^2 as follows:

$$\sum_k \Rightarrow \int dm^2 \, \frac{R^n m^{n-2}}{(4\pi)^{n/2}\Gamma(n/2)} \, . \tag{9}$$

The size R of the extra dimension, the scale M_S and G_N are related by [1]

$$G_N R^n M_S^{n+2} = (4\pi)^{\frac{n}{2}} \Gamma(n/2) \, . \tag{10}$$

For the similar process $e^+e^- \to \gamma G$ we can reproduce the expression given in Ref. [9] (for unpolarized case):

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha G_N}{4\left(1-\frac{m^2}{s}\right)} & \left[(1+\cos^2\theta)\left(1+(\frac{m^2}{s})^4\right)\right. \\ & \left. + \left(\frac{1-3\cos^2\theta+4\cos^4\theta}{1-\cos^2\theta}\right)\frac{m^2}{s}\left(1+(\frac{m^2}{s})^2\right) + 6\cos^2\theta(\frac{m^2}{s})^2\right] \, . \end{aligned} \tag{11}$$

III. CROSS SECTIONS AND DISTRIBUTIONS

A. $e^+e^- \to ZG$

We start with the result for the associated production of graviton with the Z. The total cross sections for the signal ($e^+e^- \to ZG$) and background ($e^+e^- \to Z\nu\bar{\nu}$) versus $\sqrt{s}$ for $n = 2$ and $M_S = 2.5, 4$ TeV with an angular cut $|\cos\theta_Z| < 0.8$ are shown in Fig. 1, where we also show the bench-mark process $e^+e^- \to ZH$. The angular cut, though is not necessary, can help reducing the background. The background contains three flavors of neutrinos. The $Z\nu_\mu\bar{\nu}_\mu$ and $Z\nu_\tau\bar{\nu}_\tau$ production mainly comes from ZZ production and decreases with $\sqrt{s}$, while $Z\nu_e\bar{\nu}_e$ can also come from the $t-$channel W-exchange diagrams and so increases with $\sqrt{s}$. The signal cross section increases with $\sqrt{s}$ because more KK levels contribute to the cross section. Due to phase space suppression the signal cross section is rather small at low $\sqrt{s}$. Only until $\sqrt{s}$ reaches at least 0.5 TeV does the signal become significant relative to the background.

We look more closely at a particular $\sqrt{s}$ and examine kinematic distributions and see if we can find some ways to improve the signal-to-background ratio. We choose $\sqrt{s} = 1$ TeV, $n = 2$, and $M_S = 2.5$ TeV. The ZZ production is rather back-to-back while the ZG is less back-to-back, and that is why we imposed a cut on the angle of the Z boson, which will not hurt the signal too much. In reality, the Z boson actually decays visibly into either a pair of quarks or leptons with a branching ratio of 0.8. Experimental reconstruction of the Z boson is excellent and so in this study we only impose a smearing on the Z momentum to approximate the decay. We have used an energy resolution of $\delta E/E = 0.2/\sqrt{E}$ for the Z boson, which gives approximately a 15–20 GeV spread of the reconstructed Z mass. In Fig. 2 we use $M_S = 2.5$ TeV. In Ref. [6], the effective theory with a cut-off at M_S remains valid up to a few times of M_S as long as unitarity is concerned. Perhaps, the gravitation interaction already becomes strong at or below the scale M_S.

TABLE I. The signal S, background B, signal-to-background ratio S/B, and the significance $S\sqrt{\mathcal{L}}/\sqrt{B}$ for $e^+e^- \to ZG$ at e^+e^- colliders of various $\sqrt{s}$ with a luminosity $\mathcal{L}$ of 50 fb^{-1} for $n = 2$ and $M_S = 2.5$ TeV. Cuts of $|\cos\theta_Z| < 0.8$ and $M_{\rm recoil} > 200$ GeV are imposed.

$\sqrt{s}$ (TeV)	S (fb)	B (fb)	S/B	$S\sqrt{\mathcal{L}}/\sqrt{B}$
0.5	13.1	179	0.073	6.9
0.75	48.9	334	0.15	18.9
1.0	115	452	0.25	38.1
1.25	217	543	0.40	65.8
1.5	360	613	0.59	103

[1] The definition of the cut-off scale M_S (we follow Ref. [8]) is different from the cut-off scale M of Ref. [9]. M_S is related to the M by $M_S^4 = 4M^4$ for $n = 2$.

Another useful variable is the recoil mass, $M_{\rm recoil}$, which is defined as

$$M_{\rm recoil} = \left[s - 2\sqrt{s}E_Z + m_Z^2(\text{recons.}) \right]^{1/2} . \quad (12)$$

In Fig. 2(a) we compare the recoil mass spectrum of the signal with that of the background. Obviously, a part of the background comes from ZZ production and, therefore, the visible Z boson recoiled against the another Z boson. It explains a peak around 90 GeV. The spectrum for the signal, on the other hand, does not show any peak structure. This is a characteristics of the continuous mass spectrum of the KK levels. A cut of

$$M_{\rm recoil} > 200 \;\; \text{GeV} \quad (13)$$

can remove the ZZ background. However, the rest of the recoil mass spectra for the signal and background look very much alike. Another useful distribution is the transverse momentum of the visible Z boson. The background falls more rapidly than the signal, which means we can impose a cut on the transverse momentum to increase the signal-to-background ratio: see Fig. 2(b). From Fig. 2(b) a cut of about 150 GeV may help but, however, it also cuts away about half of the signal. Therefore, we do not impose any cuts on p_{T_Z}.

TABLE II. Table showing the limit of M_S that can be reached at e^+e^- colliders of various $\sqrt{s}$. The signal S, S/B, and the significance $S\sqrt{\mathcal{L}}/\sqrt{B}$ are also shown. A luminosity $\mathcal{L}$ of 50 fb^{-1} is assumed. Part (a) requires both the significance larger than 5 and $S/B > 0.1$ for discovery while part (b) only requires significance larger than 5. Cuts of $|\cos\theta_Z| < 0.8$ and $M_{\rm recoil} > 200$ GeV are imposed.

$\sqrt{s}$ (TeV)	M_S limit (TeV)	S (fb)	S/B	$S\sqrt{\mathcal{L}}/\sqrt{B}$
(a) With $S/B > 0.1$ and $S\sqrt{\mathcal{L}}/\sqrt{B} > 5$				
0.5	2.3	17.9	0.1	9.5
0.75	2.8	33.4	0.1	12.9
1.0	3.2	45.2	0.1	15.0
1.25	3.5	54.3	0.1	16.5
1.5	3.9	61.3	0.1	17.5
(b) With $S\sqrt{\mathcal{L}}/\sqrt{B} > 5$				
0.5	2.7	9.5	0.053	5
0.75	3.5	12.9	0.039	5
1.0	4.2	15.0	0.033	5
1.25	4.8	16.5	0.030	5
1.5	5.3	17.5	0.029	5

We show the cross sections for the signal S and the background B, the signal-to-background ratio S/B, and the significance $S\sqrt{\mathcal{L}}/\sqrt{B}$ at e^+e^- colliders of energies from 0.5 to 1.5 TeV, with the choice of $n = 2$ and $M_S = 2.5$ TeV, in Table I. The nominal yearly luminosity at these next-linear colliders is of the order of 50 fb^{-1}. With such a luminosity a decent amount of signal with a large significance is achievable for $n = 2$ and $M_S = 2.5$ TeV. Since the cross section for the signal scales as $1/M_S^4$ for $n = 2$ and $1/M_S^6$ for $n = 4$, and even higher power of $1/M_S$ for larger n, so the signal cross section drops rapidly with n or M_S. For $n > 2$ the signal cross section in Table I, being further down by some orders of magnitudes, becomes phenomenologically uninteresting. Thus, we concentrate on the case $n = 2$. In Table II, we show the limit on the cut-off scale M_S that can be obtained by requiring both the $S/B > 0.1$ and significance larger than 5. The limit ranges from about 2.3 TeV to about 4 TeV. If we only require the significance of the signal larger than 5 to set the limit, the limit on M_S is slightly better, ranging from 2.7 TeV to about 5.3 TeV. However, in this case the signal-to-background ratio becomes too small.

There are two other important backgrounds, which are $e^+e^- \to W^+W^-$ and $W^\pm\ell^\mp\nu$ production when the W boson(s) decays leptonically. If the invariant mass of the lepton pair falls within the Z mass region, such events will look like a Z boson with missing energies. Fortunately, these two backgrounds are reducible if (i) we use only the hadronic decay of the Z boson in our signal, or (ii) restrict the reconstructed Z mass to a narrower range. The semi-leptonic decay mode of $WW \to q\bar{q}'\ell\nu$ will be relevant as a true background to ZG only if the measurement of the $q\bar{q}'$ invariant mass falls within the Z mass region and the lepton ℓ is missing down the beam pipe. These two conditions reduce the $W^+W^-, W^\pm\ell^\mp\nu$ backgrounds to a negligible level. Thus, the remaining concern is the leptonic decay mode of W^+W^- and $W^\pm\ell^\mp\nu$ if we include the leptonic mode of Z in our signal. We performed a calculation of $e^+e^- \to W^\pm\ell^\mp\nu$ with $W^\pm \to \ell^\pm\nu$ ($\ell = e, \mu$), and impose the requirement that the invariant mass of $\ell^+\ell^-$ falls within the Z mass region (80–120 GeV) and impose also the other cuts: $|\cos\theta_Z| < 0.8$, $M_{\rm recoil} > 200$ GeV. We found that these $W^+W^-, W^\pm\ell^\mp\nu$ backgrounds are only about 5% of the $Z\nu\bar{\nu}$ background at $\sqrt{s} = 0.5$ TeV and continuously decreases to 0.5% at $\sqrt{s} = 1$ TeV and finally less than 0.1% at $\sqrt{s} = 1.5$ TeV. We can, therefore, safely ignore these backgrounds.

B. $e^+e^- \to \gamma G$

Here we repeat the study on the process $e^+e^- \to \gamma G$ with the background $e^+e^- \to \gamma\nu\bar{\nu}$. The cross sections for the signal and background versus $\sqrt{s}$ for $n = 2$ and $M_S = 2.5$ TeV with $|\cos\theta_\gamma| < 0.9$ (0.8) and $E_\gamma > 10$ GeV are shown in Fig. 3. The signature of the event is a single photon with missing energy in the final state. For this signature the signal cross section easily becomes larger than the background cross section when $\sqrt{s}$ reaches about 1 TeV. Again, we show the recoil mass and photon transverse momentum spectrum for the signal and the background. They show similar characteristics as the ZG channel. In table III, we show the signal and background cross sections for $n = 2$ and $M_S = 2.5$ TeV at energies from 189 GeV to 1.5 TeV. At LEPII 189 GeV energy, the signal is only about 3.2% of the background and has a significance of only 0.65. Therefore, any effect of the new gravity interactions with M_S of the order of 2.5 TeV is unnoticeable. Finally, in table IV we show the limit of M_S that can be obtained for $\sqrt{s} = 189$ GeV to 1.5 TeV, by requiring both $S/B > 0.1$ and the significance larger than 5. The limit that can be obtained at LEPII is only about 1.5 TeV while at $\sqrt{s} = 1.5$ TeV it can reach up to about 5.6 TeV.

So overall, the limit on the cut-off scale M_S ranges from about 1.5 TeV to 5.6 TeV for $\sqrt{s}$ from LEPII energy to 1.5 TeV at e^+e^- colliders using the associated production of graviton with a Z boson or a photon. Such a small increase in the limit is due to the scaling of the cross section with high powers of $1/M_S$.

TABLE III. The signal S, background B, signal-to-background ratio S/B, and the significance $S\sqrt{\mathcal{L}}/\sqrt{B}$ for $e^+e^- \to \gamma G$ at e^+e^- colliders of various $\sqrt{s}$ and a luminosity $\mathcal{L}$ of 50 fb^{-1} (0.5 fb^{-1} at LEPII) for $n = 2$ and $M_S = 2.5$ TeV. Cuts of $|\cos\theta_\gamma| < 0.9$, $E_\gamma > 10$ GeV, and $M_{\rm recoil} > 200$ GeV (120 GeV at LEPII and $\sqrt{s} = 0.25$ TeV) are imposed.

$\sqrt{s}$ (TeV)	S (fb)	B (fb)	S/B	$S\sqrt{\mathcal{L}}/\sqrt{B}$
0.189	26.2	815	0.032	0.65
0.25	59.5	972	0.061	13.5
0.5	352	1406	0.25	66.5
0.75	955	1652	0.58	166
1.0	1898	1791	1.06	317
1.25	3211	1878	1.71	524
1.5	4912	1939	2.53	789

TABLE IV. Table showing the limit of M_S that can be reached at e^+e^- colliders of various $\sqrt{s}$ for the process $e^+e^- \to \gamma G$. The signal S, S/B, and the significance $S\sqrt{\mathcal{L}}/\sqrt{B}$ are also shown. A luminosity $\mathcal{L}$ of 50 fb^{-1} (0.5 fb^{-1} at LEPII) is assumed. It requires both the significance larger than 5 and $S/B > 0.1$ for discovery. Cuts of $|\cos\theta_\gamma| < 0.9$, $E_\gamma > 10$ GeV, and $M_{\rm recoil} > 200$ GeV (120 GeV at LEPII and $\sqrt{s} = 0.25$ TeV) are imposed.

(a) With $S/B > 0.1$ and $S\sqrt{\mathcal{L}}/\sqrt{B} > 5$

$\sqrt{s}$ (TeV)	M_S limit (TeV)	S (fb)	S/B	$S\sqrt{\mathcal{L}}/\sqrt{B}$
0.189	1.5	202	0.25	5
0.25	2.2	97.3	0.1	22.0
0.5	3.1	141	0.1	26.6
0.75	3.9	165	0.1	28.7
1.0	4.5	179	0.1	29.9
1.25	5.1	188	0.1	30.6
1.5	5.6	194	0.1	31.1

IV. COMPARISON WITH EXISTING DATA

A. $e^+e^- \to ZG$

As in the last section, we put in the cross section for the process $e^+e^- \to ZH$ as a comparison to the signal of $e^+e^- \to ZG$. These two processes share the same signature when the Higgs boson decays invisibly [12]. Recently, L3, ALEPH, and DELPHI [11] have searched for the invisibly decaying Higgs in association with a Z boson. We shall use their data to constrain the cut-off scale M_S.

The best limit on M_H with the Higgs boson decaying invisibly was obtained by ALEPH [11] in their recent analysis at LEPII with $\sqrt{s} = 189$ GeV. The 95%CL lower limit on M_H is 87.5 GeV (only preliminary) assuming the Higgs boson is produced with the SM strength in the ZH production and the Higgs boson decays 100% invisibly. We calculate the corresponding 95%CL upper limit on the cross section $\sigma(e^+e^- \to ZH) = 0.55$ pb in the leading-order. With the following two approximations we can apply this limit directly to $e^+e^- \to ZG$: (i) since the reported selection efficiencies are rather stable for a fairly wide range of M_H (we refer to the similar analysis done at $\sqrt{s} = 181 - 184$ GeV, the details at $\sqrt{s} = 189$ GeV are not available), we can treat the limit of $\sigma(e^+e^- \to ZH) = 0.55$ pb as roughly constant over a wide range of M_H. (ii) The signal of $e^+e^- \to ZG$ has the same signature as $e^+e^- \to ZH$ and so they should have similar efficiencies. We can then apply this cross section limit to the signal of ZG and we obtain the 95%CL limit on M_S:

$$M_S \gtrsim 515 \text{ GeV} . \tag{14}$$

This estimate is only a rough estimate but should be sufficient for our purpose. Even if there were a large difference between the selection efficiencies for ZG and ZH (which is not likely), the change in M_S would still be small, because the cross section scales as $1/M_S^4$ for $n = 2$.

B. $e^+e^- \to \gamma G$

The LEP Collaborations [13] have been searching for single-photon events with missing energies. This is an interesting signature for a number of new physics, including supersymmetry, which are (i) $e^+e^- \to XY, X \to Y\gamma$, (ii) $e^+e^- \to \tilde{G}\tilde{G}\gamma$, and (iii) $e^+e^- \to \tilde{G}\tilde{\chi}_2^0, \tilde{\chi}_2^0 \to \tilde{\chi}_1^0\gamma$. Their limits on the single-photon cross section are rather model-dependent because the detection efficiencies depend on model parameters. Since in our case it is very difficult to fully simulate the experimental conditions, we simply use their 95%CL upper limits on production cross sections. It means we assume that the detection efficiencies for our graviton signal are within the ranges of these experiments. We show the limits on the cut-off scale M_S in table V. In the table, we can see that the limits reported by the LEP Collaborations have rather wide ranges, simply because of the wide range of the detection efficiencies. It justifies our assumption that the efficiencies of our graviton signal are easily within the ranges of these LEP experiments. The limits we obtain are from about 1.2 to 2.2 TeV (it is consistent with the value of $M = 1.2$ TeV obtained in Ref. [9], please see the footnote.)

TABLE V. The range of the 95%CL upper limit on the single photon production cross section at $\sqrt{s} = 183$ GeV (the data by DELPHI is preliminary at $\sqrt{s} = 189$ GeV) given by the LEP Collaborations and the corresponding limits of the cut-off scale obtained from these measurements.

95%CL limit of single-photon cross section	95%CL limit on M_S
L3 $E\gamma > 5$ GeV, $\lvert\cos\theta_\gamma\rvert < 0.97$, $\sigma_{95} \sim 0.1 - 0.5$ pb	$\sim 1.5 - 2.2$ TeV
ALEPH $p_{T_\gamma} > 0.0375\sqrt{s}$, $\lvert\cos\theta_\gamma\rvert < 0.95$, $\sigma_{95} \sim 0.1 - 0.6$ pb	$\sim 1.2 - 1.9$ TeV
OPAL $x_T = p_{T_\gamma}/E_{\rm beam} > 0.05$, $\lvert\cos\theta_\gamma\rvert < 0.966$, $\sigma_{95} \sim 0.075 - 0.8$ pb	$\sim 1.2 - 2.2$ TeV
DELPHI $x_\gamma = E_\gamma/E_{\rm beam} > 0.06$, $45^\circ < \theta_\gamma < 135^\circ$, $\sigma_{95} \sim 0.3 - 0.4$ pb	$\sim 1.25 - 1.35$ TeV

V. CONCLUSIONS

Excess signals of missing energy events in the process $e^+e^- \to Z(\gamma) + \not{E}$ can provide a useful test for the low scale gravity with extra space dimension compactified at the size of mm. This suggests a TeV e^+e^- collider can investigate the direct graviton production and study the early unification. Using the available data at LEPII a limit of $M_S \gtrsim 515$ GeV is obtained in the ZG channel while $M_S \gtrsim 1.2 - 2.2$ TeV is obtained in the γG channel. For the future e^+e^- linear colliders of energies $0.25 - 1.5$ TeV and a luminosity of 50 fb^{-1} M_S can be probed up to about 5.6 TeV. Although some other non-accelerator physics, e.g., cooling of supernova [5], may give a much stronger constraint on the cut-off scale M_S, the collider signatures, including those considered in this paper, would provide independent tests for the new gravity interactions.

ACKNOWLEDGMENTS

This research was supported in part by the U.S. Department of Energy under Grants Nos. DE-FG02-84ER40173 and DE-FG03-91ER40674 and by the Davis Institute for High Energy Physics. K.C. wants to thank the National Center for Theoretical Sciences in Taiwan where part of this work was done.

---

[1] P. Horava and E. Witten, Nucl. Phys. **B460**, 506 (1996); *ibid.*, **B475**, 94 (1996); E. Witten, *ibid.* **B471**, 135 (1996).

[2] See for example, T. Li, J. Lopez, D. Nanopoulos, Phys. Rev. **D56**, 2602 (1997); E. Dudas, C. Grojean, Nucl. Phys. **B507**, 553 (1997); J. Ellis, Z. Lalak, S. Pokorski, and W. Pokorski, Nucl.Phys. **B540**, 149 (1999).

[3] See for example, P. Horava, Phys. Rev. **D54**, 7561 (1996); H. Nilles, M. Olechowski, M. Yamaguchi, Nucl. Phys. **B530**, 43 (1998); E. Mirabelli and M. Peskin, Phys. Rev. **D58**, 065002 (1998); L. Randall and R. Sundrum, e-Print archive: hep-th/9810155.

[4] I. Antoniadia, Phys. Lett. **B246**, 377 (1990); J. Lykken, Phys. Rev. **D54**, 3693 (1996); G. Shiu and S. Tye, Phys. Rev. **D58**, 106007 (1998); I. Antoniadis and C. Bachas, e-Print Archive hep-th/9812093.

[5] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, G. Dvali, SLAC-PUB-7864, e-Print Archive: hep-ph/9807344; N. Arkani-Hamed, S. Dimopoulos, J. March-Russell, SLAC-PUB-7949, e-Print Archive: hep-th/9809124.

[6] G. Giudice, R. Rattazzi, J. Wells, CERN-TH-98-354, e-Print Archive: hep-ph/9811291.

[7] S. Nussinov and R. Shrock, e-Print archive hep-ph/9811323.

[8] T. Han, J. Lykken, and R. Zhang, e-Print Archive: hep-ph/9811350.

[9] E. Mirabelli, M. Perelstein, and M. Peskin, SLAC-PUB-8002, e-print hep-ph/9811337.

[10] J. Hewett, SLAC-PUB-8001, e-Print Archive: hep-ph/98011356; T. Rizzo, SLAC-PUB-8036, e-Print Archive: hep-ph/9901209; P. Mathews, S. Raychaudhuri, and K. Sridhar, TIFR-TH-98-51, e-Print Archive: hep-ph/9812486; P. Mathews, S. Raychaudhuri, and K. Sridhar, TIFR-TH-98-48, e-Print Archive: hep-ph/9811501; Z. Berezhiani and G. Dvali, NYU-TH-10-98-06, e-Print Archive: hep-ph/9811378; K. Agashe and N. Deshpande, e-Print Archive: hep-ph/9902263; M. Graesser, e-Print Archive: hep-ph/9902310; P. Nath and M. Yamagchi, e-Print Archive: hep-ph/9902323.

[11] L3 Coll., Phys. Lett. **B418**, 389 (1998); ALEPH Coll., CERN-EP/99-008, Jan. 1999; DELPHI Coll., submission to ICHEP'98 Conference, Vancouver, DELPHI 98-126 CONF 187; ALEPH Coll., CERN seminar at LEPC, Nov. 1998, available at the ALEPH website.

[12] F. de Campos, O. Eboli, J. Rosiek, J.W.F. Valle, Phys. Rev. **D55**, 1316 (1997); O. Eboli, et al. Nucl. Phys. **B421**, 65 (1994).

[13] ALEPH Coll., Phys. Lett. **B429**, 201 (1998); L3 Coll., Phys. Lett. **B444**, 503 (1998); OPAL Coll., e-Print archive, hep-ex/9810021; DELPHI Coll., submission to the ICHEP'98 Conference, Vancouver, DELPHI-98076 CONF 144; DELPHI Coll., CERN seminar by V. Ruhlmann-Kleider at LEPC, Nov 1998.

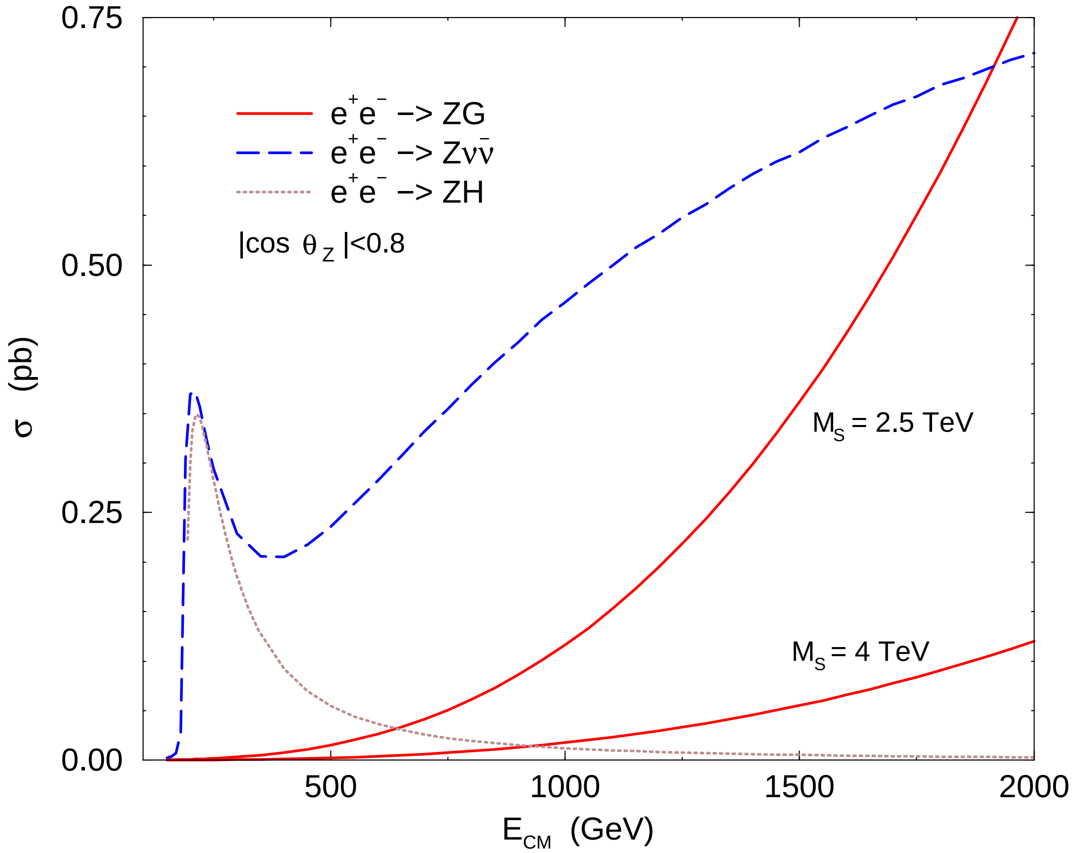
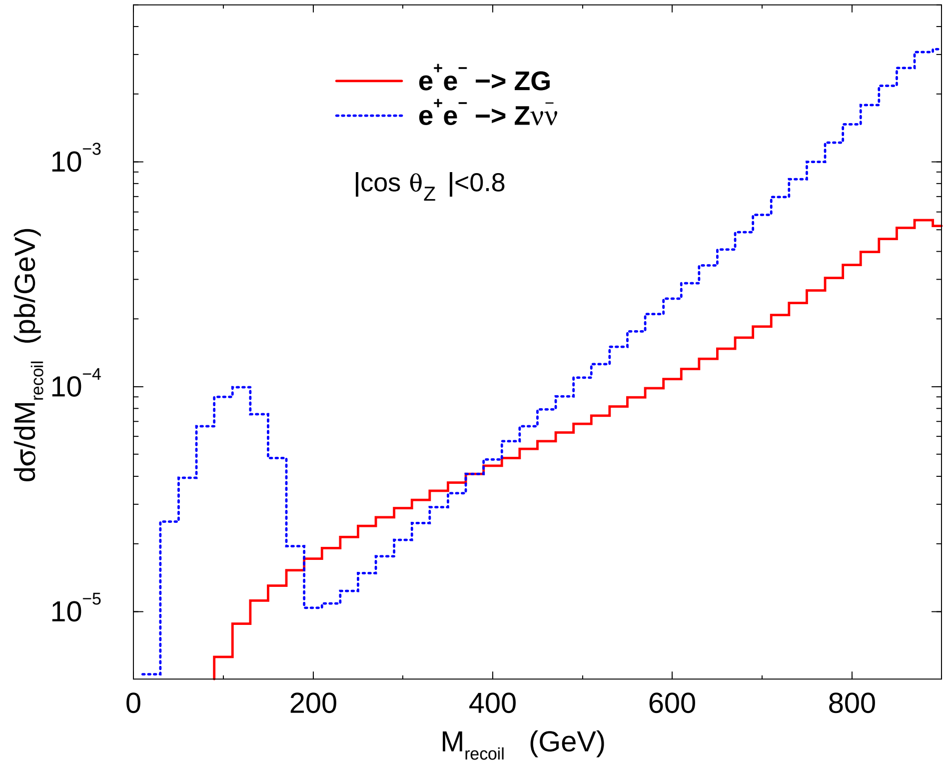


FIG. 1. The total cross sections for $e^+e^- \to Z\nu_i\bar{\nu}_i (i = e, \mu, \tau)$, $e^+e^- \to ZG$, and the $e^+e^- \to ZH$ with $|\cos\theta_Z| < 0.8$. We used $n = 2$ and $M_S = 2.5$ and 4 TeV as shown.



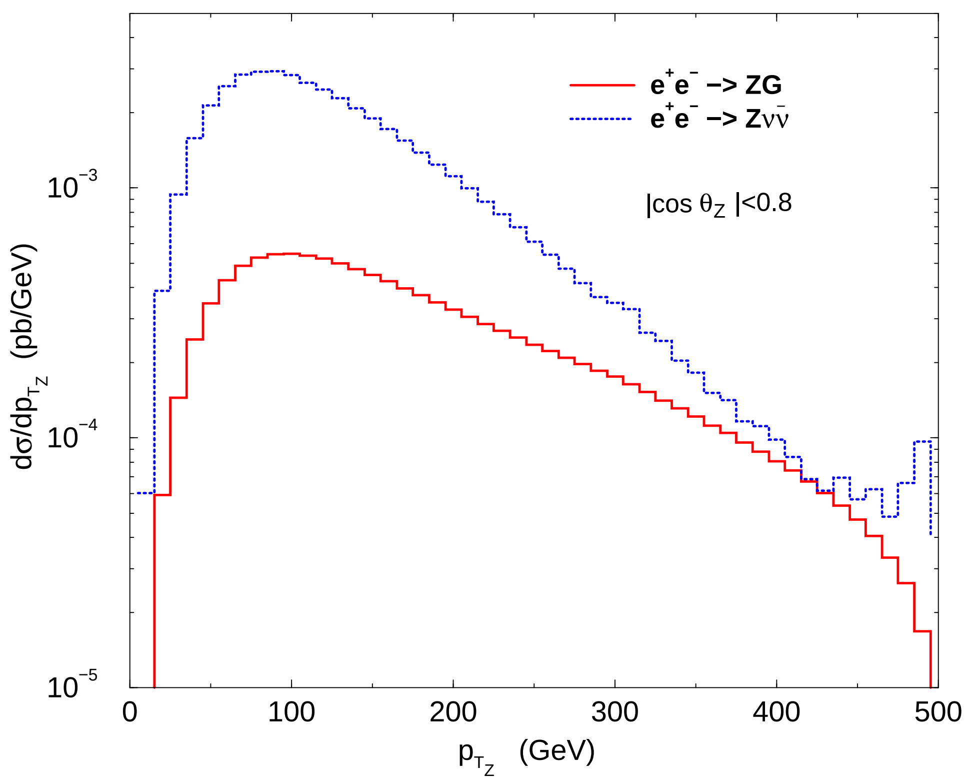


FIG. 2. (a) The differential distribution $d\sigma/dM_{\rm recoil}$ and (b) $d\sigma/dP_{T_Z}$ for $e^+e^- \to Z\nu_i\bar{\nu}_i$ $(i = e, \mu, \tau)$ and $e^+e^- \to ZG$ for $n = 2$ and $M_S = 2.5$ TeV. We imposed a cut of $|\cos\theta_Z| < 0.8$.

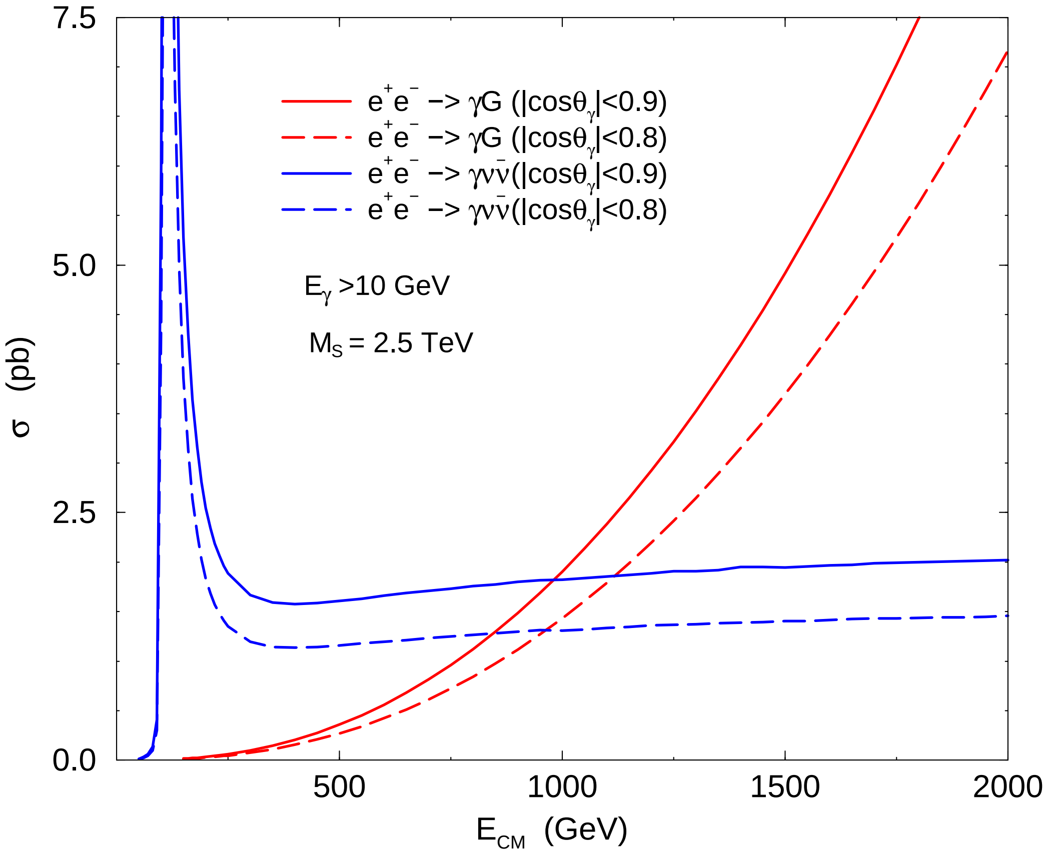


FIG. 3. The total cross sections for $e^+e^- \to \gamma\nu_i\bar{\nu}_i (i = e, \mu, \tau)$ and $e^+e^- \to \gamma G$ with cuts as shown. We used $n = 2$ and $M_S = 2.5$ TeV.

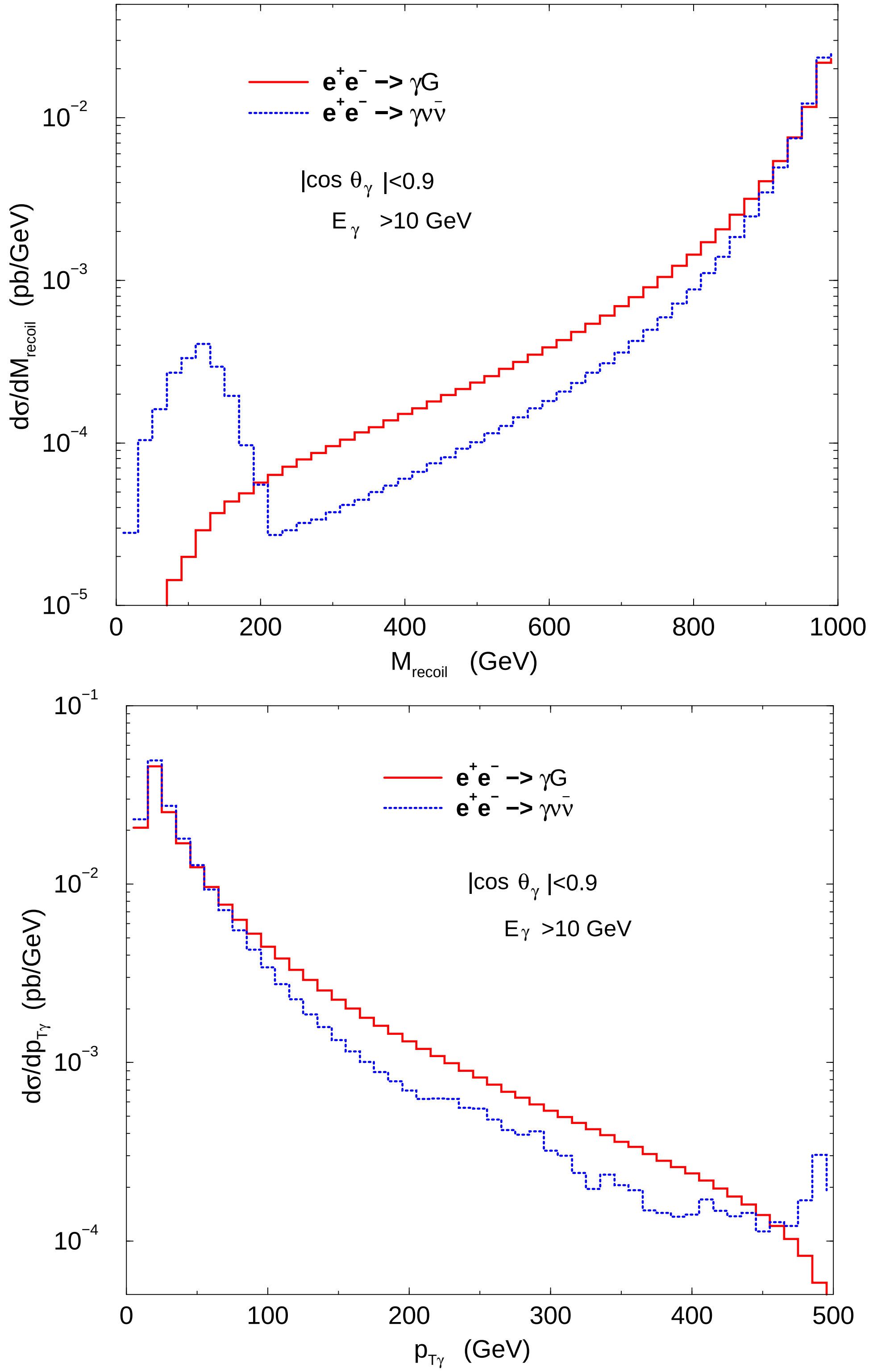


FIG. 4. (a) The differential distribution $d\sigma/dM_{\rm recoil}$ and (b) $d\sigma/dP_{T_\gamma}$ for $e^+e^- \to \gamma\nu_i\bar{\nu}_i$ ($i = e, \mu, \tau$) and $e^+e^- \to \gamma G$ for $n = 2$ and $M_S = 2.5$ TeV. We imposed cuts of $|\cos\theta_\gamma| < 0.9$ and $E_\gamma > 10$ GeV.